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MEASUREMENT OF CONDITIONALLY AVERAGED TURBULENCE CHARACTERISTICS IN THE PLANE WAKE BEHIND A CYLINDER

A. A. Praskovskii

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Methods describing turbulent flows using equations for probability density distribution (PDD) for velocity and concentration fluctuations are being actively developed [1, 2] in recent times. Such an approach to the study of turbulence is especially fruitful for the analysis of flows with chemical reactions. In formulating the closure of the equations for PDD, certain hypotheses based on the physical characteristics are used that require experimental verification. In particular, in obtaining the closure of equations for PDD of velocity fluctuations [1] on the basis of the Kolmogorov-Obukhov theory [3], it has been hypothesized that in a turbulent flow, turbulent energy dissipation measured at a constant value of velocity, does not depend on this value. Measurements of the dispersion of the streamwise velocity gradient in the plane of symmetry in the wake behind a circular cylinder where the flow is fully turbulent have demonstrated the correctness of this hypothesis [4]. The objective of the present paper is to verify the hypothesis given in [1] in those regions of turbulent flow where the intermittency coefficient is different from one, while the results of the measurements of the dispersion of the time derivative of streamwise velocity are used to estimate turbulent energy dissipation. During the measurements, a number of other conditionally averaged turbulent flow characteristics have been obtained which are of independent interest and some of them are also presented in this paper.

1. Measurements were made in the plane wake behind a circular cylinder of diameter d = 36 mm at a relative distance x/d = 38.6 behind the cylinder. The cylinder was mounted at a nozzle section of diameter 1200 mm in a wind-tunnel with open test-section, the freestream turbulence in the absence of the cylinder was 0.4% at the nozzle section and 0.6% at the measuring section. Tests were conducted at a velocity $U_0 = 5.24$ m/sec, which corresponds to a Reynolds number Re = $U_od/v = 1.26 \cdot 10^4$, where v is the kinematic coefficient of viscosity. Constant-temperature hot-wire anemometer DISA 55A01 with the transducer 55A22 using platinized tungsten wire, 5 µm in diameter and 1 mm long, was used to measure streamwise mean velocity component U and velocity fluctuations u(t), where t is the time. The output signal was recorded in the measuring ChM magnetometer "MR 800A Labcorder" in the frequency range 0-5 kHz, the recording time for each frame was 45 sec. The recorded realizations were passed through filters with a characteristic slope of 48 dB/octave and the lower and upper frequency bounds $f_1 = 1$ Hz and $f_{11} = 800$ Hz, respectively, and then in frequency sampling of analog digital converter $f_0 = 5$ kHz they were fed to a computer where their statistical characteristics were computed. The limitation of the frequency range of fluctuations in the high-frequency region made it possible to ensure a signal-to-noise ratio of 39-43 dB, but led to a reduction in the values of dispersion of the velocity gradient (quantitative estimates are given below) in the tests, and the energy spectrum of fluctuating velocities rapidly falls with an increase in frequency. Hence there is always a certain frequency, approximately equal to 2 kHz in the given experiment, at which the spectral density of the signal and noise are equalled and above which the noise exceeds the signal. At 800 Hz, the signal level was an order of magnitude higher than the noise and it determined the choice of the frequency limit for the filter. It is worth noting that in these experiments the basic source of noise was the magnetograph whose characteristic dynamic range at $f_u = 5 \text{ kHz}$ was approximately 37 dB.

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The method of computing unconditionally averaged moments and PDD of random processes are quite well known (see, e.g., [5]), and there is no need to consider it here. In order to compute turbulence characteristics averaged for turbulent fluid, as it is well known [6], an intermittency function I(t) is introduced which is equal to one when turbulent fluid is present at the observation points and equals zero in the absence of turbulence. Then the value of n-th-order moment of velocity fluctuations averaged, e.g., in relation to the fluid turbulence, is computed from [6]

 $\langle u^n(t) \rangle_t = \langle u^n(t)I(t) \rangle / \gamma_I,$

where $\gamma_I = \langle I(t) \rangle$ is the intermittency coefficient obtained by averaging intermittency function; the index t indicates that averaging is done with respect to fluid turbulence and the brackets $\langle \rangle$ indicate averaging by time.

The algorithm for the determination of the intermittency function is sufficiently well developed (see, e.g., [7]). In order to measure I(t) the so-called detector function $\phi(t) \ge$ 0, directly related to vortex fluctuations in the flow, is first chosen. (We draw attention to the fact that turbulent fluid differs from nonturbulent fluid only by the presence of vortex fluctuations, since velocity fluctuations are present even in nonturbulent flow.) Functions $\left|\frac{\partial u}{\partial t}\right|$, $\left|\frac{\partial^2 u}{\partial t^2}\right|$, etc. (see [7]) are generally used as $\varphi(t)$ in measurement with single wire sensor. It is further assumed that I(t) = 1 when $\varphi(t) \ge H$ and I(t) = 0 when $\varphi(t) < H$, where H is a threshold value chosen in some manner. However, owing to fluctuations in the detector function at any threshold level the condition $\varphi(t) < H$ is satisfied during short intervals of time even inside turbulent fluid which leads to the appearance of "parasitic" zeros [i.e., of I(t) = 0 inside turbulent fluid] in the intermittency function. In order to eliminate "parasitic" zeros in I(t) before the comparison with the threshold level, the detector function is passed through smoothing (integrating) filter with limiting frequency ff. This process should not be equated to filtration in the region of high-frequency velocity fluctuations u or its derivative $\partial u/\partial t$. The purpose of smoothing positively determined detector function is to maintain the signal $\varphi(t)$ above the threshold level H for a characteristic time of the order $1/2\pi f_f$ inside the turbulent fluid.

A large number of studies are available where different aspects of the measurement of the detector function are considered (see, e.g., [7, 8]), but there has been no study so far to develop a unique approach to study the problem of the selection of the detector function, limiting frequency of the smoothing filter, and the threshold level. This is mainly because there is no sharp boundary between turbulent and nonturbulent fluids at finite Reynolds numbers. Consequently, there is always an arbitrariness in the measurement of intermittency function and the error in the measurement of intermittency coefficient is on the order of $\operatorname{Re}^{-1/4}_{L}$ (see [1, 4]), where $\operatorname{Re}_{L} = u'L/v$ is the turbulence Reynolds number, $u' = \sqrt{\langle u^{2}(t) \rangle}$ is the rms value of the fluctuations, L is an integral scale. Detector functions $\varphi(t) = |u|, |\partial u|$ $\partial t|$, $|\partial^2 u/\partial t^2|$, $(\partial u/\partial t)^2$, $(\partial^2 u/\partial t^2)^2$ were considered for a valid choice of the parameters $\varphi(t)$, ff, and H under the experimental conditions at four points of the section in which the values of the intermittency coefficient γ_e obtained from the excess of the derivative of velocity fluctuations [6] were $\gamma_e = 1$, 0.698, 0.335, and 0.0706. Dependence $\gamma_I(H)$ was determined for each of these functions at different values of ff. The analysis of these relations made it possible to express the function $\varphi(t)$ and the limiting frequency for the smoothing filter ff for which the function $\gamma_{I}(H)$ is relatively weakly dependent on the threshold level when $\gamma_{I}(H) \approx \gamma_{e}$. Experiments showed that of the detector functions considered here, the function $\varphi(t) = |\partial u/\partial t|$ best satisfies this condition when $f_{f} = 0.23 [\langle (\partial u/\partial t)^{2} \rangle / \langle u^{2} \rangle]^{1/2}$. Here the range of threshold levels for which $\gamma_{I}(H) \approx \gamma_{e}$ was $H = (0.25-0.32) < |\partial u/\partial t| > /\gamma_{e}$. In general, the quantity ff should be associated with the characteristic frequency $[<(\partial^{2}u/\partial t^{2})^{2}>/<(\partial u/\partial t)^{2}>]^{1/2}$ and not with $[<(\partial u/\partial t)^{2}>/<u^{2}>]^{1/2}$, since only the former determines, on the average, the number of zeros of the function $|\partial u/\partial t|$ in unit time [9], but it could be expected that at moderate values of Reynolds numbers these characteristic frequencies are of one order, which is confirmed by the measurements. Finally, $\varphi(t) = |\partial u/\partial t|$, $f_f = 0.23$ [<($\partial u/\partial t$)²>/< u^2 >]^{1/2} (which was approximately 140 Hz), H = 0.28 <| $\partial u/\partial t$ |>/ γ_e were used in this study to compute the intermittency function.

Taylor's hypothesis $\partial u/\partial x = (1/U)\partial u/\partial t$, which is well suited for wake-type flow, was used to estimate turbulent energy dissipation. The dispersion of the time derivative of velocity fluctuations $\langle (\partial u/\partial t)^2 \rangle$ determined in the experiments was uniquely associated with the energy dissipation only in the case of isotropic turbulence. Although turbulence in the wake of a cylinder is not isotropic, the quantity $\langle \varepsilon \rangle = 15v \langle (\partial u/\partial x)^2 \rangle$ can be considered as an approximate estimate of turbulent energy dissipation. Indicator function $\delta(t)$ equal to one when $u_* - \Delta u \le u(t) \le u_* + \Delta u$, and zero at other times, was constructed to compute conditional mean value of the dispersion of the derivative $\langle (\partial u/\partial t)^2 \rangle_{u_*}$ at a given level of fluctuation velocity $u(t) = u_*$

$$\langle (\partial u/\partial t)^2 \rangle_{u_*} = \frac{\langle (\partial u/\partial t)^2 \,\delta(t) \rangle}{\langle \delta(t) \rangle}.$$

The conditional mean value of the dispersion of the velocity derivative at a given velocity in the turbulent fluid was computed from

$$\langle (\partial u/\partial t)^2 \rangle_{u_{*,t}} = \frac{\langle (\partial u/\partial t)^2 \delta(t) I(t) \rangle}{\langle \delta(t) I(t) \rangle}.$$

In these experiments the value of Δu was given by u = 0.05u'.

2. It is known [6] that plane wake behind a circular cylinder can be considered approximately self-similar when x/d > 100, and the complete self-similarity occurs when x/d > 1000. Flow characteristics when x/d = 38.6 are not self-similar, but in order to represent experimental results it is convenient to use similarity variable $\eta = y/l_c$, where y is the distance from the plane of symmetry of the wake, $l_c = \sqrt{(x - x_0)d}$ is the effective width, and $x_0 = -50d$ is the virtual origin of the wake, chosen in accordance with [10].

Results of measured mean velocity profiles are shown in Fig. 1, where $\Delta U_m = U_0 - U(n = 0)$ is the maximum velocity defect, being 1.16 m/sec in the given experiment, the dashed line represents similarity profile taken from [6], obtained at x/d = 500-950, Re = 1360. It is seen that functionally the shape of the profile even at x/d = 38.6 is quite close to the similarity profile and by a corresponding shift in the virtual origin x_0 , it is possible to obtain a good quantitative agreement for the above profiles. This result agrees qualitative-ly with [6, 10] and confirms that the mean flow characteristics more rapidly approach similarity form than the parameters of the fluctuating flow.

Results of measurements of the intermittency coefficient are shown in Fig. 1 as obtained by two methods: by averaging the intermittency function $\gamma_{I} = \langle I(t) \rangle$ (point 1) and from the excess coefficient of the derivative of velocity fluctuations $\gamma_{e} = E_{1}(\eta = 0)/E_{1}$ (point 2), where $E_{1} = \langle (\partial u/\partial t)^{4} \rangle / [\langle (\partial u/\partial t)^{2} \rangle]^{2}$, and also the profile $\gamma(\eta)$ taken from [6] and obtained by using the above two methods for x/d = 160 and Re = 6600 (dashed line). It is seen that when x/d = 38.6, the relative width of the zone in which the nonturbulent fluid is entrained by the turbulent flow (i.e., $\gamma > 0$) is less than the width in the similarity segment of the wake. The excellent agreement of the measured quantities γ_{I} and γ_{e} raises the hope that the inter-







mittency function I(t) as described in Sec. 1 is quite reliably measured and it can be used for the determination of conditionally averaged characteristics of velocity fluctuations in turbulent flow.

Some results from such measurements are shown in Figs. 1-4, in which the unconditionally averaged characteristics are indicated by circles and the averaged quantities based on turbulent fluid are indicated by crosses.

The unconditionally averaged relative strength of the velocity fluctuations $u'/\Delta U_m$ at x/d = 38.6 is appreciably more than that in the similarity region of the wake (dashed line represents data at x/d = 500-950, Re = 1360 [6]). It is worth noting that the rms value of u' and the turbulent energy dissipation < ϵ > (see Fig. 1), averaged in turbulent fluid, vary along the section much more weakly than the corresponding unconditionally averaged characteristics. These data confirm the conclusion made in [6] on the basis of experimental results and qualitative physical considerations that the strength and dissipation of turbulent energy are almost equal at all points of the section sufficiently far from the boundary of turbulent fluid. It is possible to verify that the unconditionally and conditionally averaged values of the dispersion of the velocity derivative obtained in the experiments satisfy the relation $<(\partial u/\partial t)^2 > \approx \gamma_{I} < (\partial u/\partial t)^2 >_t$ [6], which bears out the correctness of the measurement technique.

Consider the measurement error for the dispersion of the velocity derivative associated with the boundedness of the frequency domain of the fluctuations in the high-frequency range. The streamwise length scale L determined by unconditional averaging of energy spectrum in the experiments was practically constant when $0 \le \eta \le 0.47$. When $\eta = 0$ the integral scale was given by $L/l_c = 0.113$ (in the similarity region of the wake $L/l_c \approx 0.2$ [6]), which corresponds to a Reynolds number ReL = $1.13 \cdot 10^3$. It is not difficult to estimate that the dissipation constant $\alpha = 10\nu L < (\partial u/\partial t)^2 > /(u'^3 U^2)$ at $\eta = 0$ (i.e., in the fully turbulent region) is found to be equal to 0.296. Besides, the analysis of the data given in [11] of the experimental results for the turbulence characteristics behind cascades show that at ReL = $1.5 \cdot 10^3 - 1.3 \cdot 10^4$, the dissipation constant is $\alpha \approx 0.42 - 0.45$ and somewhat increases with a decrease in ReL [12]. Consequently, the limitation of the frequency domain of fluctuations by $f_u = 800$ Hz under the above experimental conditions leads to a reduction in the values of the dispersion of the velocity derivative by approximately 30-35%.

The unconditionally averaged and turbulence averaged normalized PDD of the fluctuating velocity u'P(u/u') and its derivative $u_1'P(u_1/u'_1)$, where $u_1 = \partial u/\partial t$, $u'_1 = \sqrt{\langle u_1^2 \rangle}$ were computed at all the observed points. The turbulence averaged PDD of velocity fluctuations were computed in the form $u'_tP(u_t/u'_t)$, where $u_t = u - \langle u \rangle_t$, since $\langle u \rangle_t < 0$ (see, e.g., [13]). The PDD of the fluctuating velocity and its time derivative at four points of the test section are illustrated in Figs. 2 and 3. The experimental results show that with a decrease in the value of intermittency coefficient the differences between unconditionally averaged and turbulence averaged PDD increase sharply. Conditionally averaged PDD practically do not vary



with a reduction in γ (when n = 0.472, $\gamma \approx 0.15$), i.e., inside the turbulent fluid the structure of the fluctuations practically does not vary along the section. With a reduction in the intermittency coefficient the unconditionally averaged PDD undergo all the more spiky shape, especially clearly expressed on the function $u'_1P(u_1/u'_1)$ (see also [13]). This is a natural result because the turbulent flow regions at the wake boundary where the intensity of velocity fluctuations and their derivatives are quite large, alternate with nonturbulent flow regions, where the velocity fluctuations are at a low level, and the derivatives $\partial u/\partial t$ are negligibly small. The difference between unconditional and turbulence averaged PDD for $\gamma \approx 1$ is very clearly illustrated in Fig. 4 by the results of the measurements of excess fluctuation velocity $E = \langle u^4 \rangle / (\langle u^2 \rangle)^2$ and its derivative $E_1 = \langle u_1^4 \rangle / (\langle u_1^2 \rangle)^2$, which show that when $\eta = 0.561$ ($\gamma \approx 0.027$), the unconditionally averaged coefficients E and E₁ attain values 9.89 and 197, respectively. The values of the same quantities obtained by turbulence averaging are practically unchanged along the section and are, respectively, Et ≈ 3 and E_1 , t ≈ 5 .

Theoretically, the presence of regions with nonturbulent fluid should lead to the appearance of unconditionally averaged PDD with sharp peaks, especially sharp for PDD of the derivatives of the fluctuations, whereas the relations given in Figs. 2 and 3 have quite wide smooth maxima near the center. Two causes have been mentioned in [4] for this smoothening: inaccurate measurements and the effect of molecular viscosity. Inaccuracy in measurements is caused by the use of finite values of the intervals $\delta u = 0.2u'$ and $\delta u_1 = 0.2u_1'$ in which the functions u'P(u/u') and $u_1'P(u_1/u_1')$ were averaged in order to compute PDD. The effect of molecular viscosity is of a more basic nature [1]. The sharp boundary between turbulent and nonturbulent fluids exists only at infinitely large Reynolds numbers. At finite values of Re the momentum transfer due to molecular viscosity leads to the appearance of transition layer of finite thickness between turbulent and nonturbulent flows in which vortex fluctuations are present.

It is worth noting that the turbulence averaged coefficient of excess of the derivative of the velocity fluctuation is approximately $E_{1,t} \approx 5$ (see Fig. 4), and, consequently, PDD of $u'_{1,t}P(u_{1,t}/u'_{1,t})$ significantly differs from the normal. This result confirms the statement made in [3] that the coefficient of the excess of the derivative of velocity fluctuations should exceed the value 3 corresponding to the normal distribution because of the intermittency of small-scale components of fluctuations which are present even in turbulent flow.

3. Measurements of conditional mean values of the dispersion of velocity derivative at a fixed level of fluctuating velocity $u(t) = u_{\star}$ were made at four points of the section at n = 0, 0.325, 0.384, and 0.443 ($\gamma_{I} = 0.996, 0.756, 0.540$, and 0.303, respectively). Results are shown in Fig. 5 in the nondimensional form

$$e(v) = \frac{\langle (\partial u/\partial t)^2 \rangle_v}{\langle (\partial u/\partial t)^2 \rangle_t}, \quad e_t(v) = \frac{\langle (\partial u/\partial t)^2 \rangle_{v,t}}{\langle (\partial u/\partial t)^2 \rangle_t}, \quad v = u_*/u'$$

[e(v) are given by points 1 and $e_t(v)$ by points 2).

When n = 0, the functions e(v) and $e_t(v)$ coincide, since $\gamma_I \approx 1$. In the plane of symmetry of the wake, the conditionally averaged dispersion of the derivative e(v) varies approximately by $\pm 20\%$ as v changes in the interval -2.5 to 2.5. We observe that the relative statistical error in the measurement of e(v) reached 25% at |v| = 2.5 due to the limitations of the setup. In view of this situation it is possible to consider that the results of measurements of conditionally averaged dispersion of the velocity derivative in the plane of symmetry of the wake confirms the hypothesis that the quantity $\langle (\partial u/\partial t)^2 \rangle_V$ does not depend on the level of v in fully turbulent flow and agrees well with the experimental results [4].

When $\gamma < 1$, the function e(v) increases monotonically with increase in the level of |v|. This is a natural result because, at small values of |v|, the quantity $\langle (\partial u/\partial t)^2 \rangle_v$ decreases on account of the entry of nonturbulent fluid at the given point where the fluctuations of the derivative $\partial u/\partial t$ are negligibly small. Hence, the occasionally used assumption that e(v) weakly depends on v [14] does not reflect reality at $\gamma < 1$. The dispersion of the derivative et(v) computed for the turbulent fluid varies appreciably less than e(v) at all points under consideration. When $\eta = 0.443$ ($\gamma_I = 0.303$) the quantity $e_t(v)$ increases nearly two times as the level |v| varies from 0 to 2.5, but in closing the equations for PPD of velocity fluctuations, these changes may be, apparently, neglected. The data shown in Fig. 5 indicate that the hypothesis put forward in [1] on the independence of conditionally averaged dissipation of turbulence energy from the level of averaging in the turbulent fluid is approximately satisfied when $0.3 \leq \gamma \leq 1$.

It is worth drawing attention to the fact that the functions e(v) and $e_t(v)$ practically coincide at large levels. This is because the intensity of velocity fluctuations in non-turbulent fluid is appreciably lower than in the turbulent fluid, and hence the function e(v) is practically completely determined at large |v| by the dispersion of the velocity derivative in the turbulent fluid. The unsymmetric nature of the functions e(v) and $e_t(v)$ in relation to the axis v = 0 is associated with the asymmetry of unconditionally averaged PDD of fluctuating velocity (see Fig. 2).

It was mentioned above that, in the present experiments at the upper frequency limits $f_u = 800$ Hz, the value of the dispersion of the derivative $\langle (\partial u/\partial t)^2 \rangle$ is lowered. We note that in [4] the relative frequency range for the apparatus was approximately two times less than that used in the present experiments. Hence it is necessary to consider the question of the influence of the limitation of the upper frequency limit on the measurement of the function e(v). In order to do this, the realization recorded in the magnetograph corresponding to $\eta = 0$ was again worked out at $f_u = 100$, 315, and 2000 Hz with frequency sampling of $f_0 = 0.5$, and 10 kHz, respectively. The signal-to-noise ratio at $f_u = 2$ kHz was 37 dB. The dissipation constant α at $f_u = 2$ kHz was 0.453, which agrees well with the data from [11]. (Direct measurements of the spectrum of the velocity derivative confirmed that the Kolmogorov frequency in the above experiments is approximately 2 kHz.)

The results of conditionally averaged dispersion of the velocity derivative e(v) are given in Fig. 6 for various values of the upper frequency limit; here the point 1 is for $f_u = 2$ kHz, 2 is for 800 Hz, 3 is for 315 Hz, and 4 is for 100 Hz. It is seen that the functional shape of the dependence e(v) at $0 \le v \le 2.5$ practically does not change in the interval 100 Hz $\le f_u \le 2$ kHz (which corresponds to the range $5.91 \le 2\pi f_u L/U \le 118$). This confirms the reliability of results given in [4] and here and the validity of conclusions made in them.

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